**14 Calculating R0 for non-randomly mixing populations**

**Overview**

In the last practical, we explored the impact of different levels of vaccination coverage in populations with different contact patterns. In both, the age-specific proportion of individuals who were susceptible in the absence of vaccination were identical. However, the vaccine coverage needed to control transmission was different because of the different values of R0 in two choices.

The first part of this practical illustrates how you would calculate the next generation matrix and part II illustrates methods for calculating R0.

**PART I: Calculating the next generation matrix**

Information required to calculate the basic reproduction number are given as below with average duration of infectious period of 11 days:

|  |  |
| --- | --- |
| Age category | Number of Susceptible\* |
| Young | 15000 |
| Middle-aged | 15000 |
| Old | 30000 |



1. How many secondary infectious persons among young individuals will occur as a result of the introduction of

i) 1 infectious young person

2.7390 people

ii) 1 infectious middle-aged person and

0.6864 people

iii) 1 infectious old person?

0.6864 people

As a result of the introduction of 1 infectious young person, 1 infectious middle-aged person, or 1 infectious old person, 1.66\*10-5\*15000\*11 = 2.7390 people, 4.16\*10-6\*15000\*11 = 0.6864 people, and 4.16\*10-6\*15000\*11 = 0.6864 young infectious people will occur respectively.

1. How many secondary infectious persons does each young, middle-aged and old infectious person generate in a totally susceptible population?

NGM =

The young, middle-aged and old infectious person generate 4.7982 infectious people, 2.7456 people, 2.7456 infectious people respectively in a totally susceptible population. It can be derived by adding the components by columns. (2.7390+0.6864+1.3728=4.7982, 0.6864+0.6864+1.3728=2.7456, 0.6864+0.6864+1.3728=2.7456)

**PART II: Calculating the basic reproduction number**

Introduced one infectious young individual into a totally susceptible population at the start to answer the question 1~4.

1. What proportion of infectious persons in the first generation are young, middle-aged and old as a result of the introduction of one infectious young individual?

proportion of infectious persons in the first generation (young): 0.5708

proportion of infectious persons in the first generation (middle): 0.1431

proportion of infectious persons in the first generation (old): 0.2861

(2.7390/4.7982 = 0.5708, 0.6864/4.7982=0.1431, 1.3728/4.7982=0.2861)

1. How many secondary infectious persons resulted directly from the initial infectious person introduced into the population?

4.7982 people becomes infectious. It is the summation of the first column of NGM shown in PART 1 because the initial infectious person introduced into the population is a young individual.

1. What happens to the age distribution of the infectious persons in each generation after a few generations have occurred?

Chart, line chart

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Proportion age distribution of the infectious persons for young, middle, old people converges to 43%, 19%, 38% each. It started from young: 57%, middle: 14%, old: 29%. Proportion of the young slowly declined and simultaneously, the proportion of the middle and old slowly increased.

1. What is the average number of secondary infectious persons resulting from each infectious person after a few generations have occurred?

The number of secondary infectious people resulting from each infectious person converges to Gk/Gk-1 = 3.6356. We can guess that the basic reproduction number(R0) will be about 3.6356.

R =

3.9173 3.7192 3.6617 3.6439 3.6383 3.6365 3.6359 3.6357 3.6356

1. Change the numbers of infectious persons introduced into the population at the start to take the following values:

i) 20, 50, 30 young, middle-aged and old infectious persons respectively.

ii) 0.5, 0.2 and 0.3 young, middle-aged and old infectious persons respectively.

How does changing the values for the numbers of infectious persons introduced into the population at the start alter your answer to Q3 and Q4?

Chart, line chart

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Graphical user interface, chart

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Whether the number of initial infectious people of the young, middle-aged, and old are [1,0,0], [20,50,30] or [0.5,0.2,0.3] respectively, they all converge to young: 43%, middle:19%, old:38%. The different initial number changes the shape of the beginning of the graph, but in the end, they both converge to the same proportion value.

Adding to that, the basic reproduction number (R0) is about 3.6356 in all 3 cases.

R (20, 50, 30) =

3.4590 3.5762 3.6163 3.6294 3.6336 3.6350 3.6354 3.6355 3.6356

R (0.5, 0.2 and 0.3) =

3.6776 3.6489 3.6398 3.6370 3.6360 3.6358 3.6357 3.6356 3.6356

1. What are the maximum eigenvalue and the corresponding eigenvector of the next generation matrix? How are these related to your answer to the previous questions?

The maximum eigenvalue of the next generation matrix is 3.6356. The normalized eigenvector is [0.4336, 0.1888, 0.3776]’. The normalized eigenvector is the same as the converging age proportion distribution of the infectious people.

1. Change the value for the proportion of immune to see what happens to the average number of secondary infectious persons resulting from each infectious person after a few

generations have occurred if the following proportions of the population are immune:

i) 25% ii) 50% iii) 72.5% ⅳ) 75%

Chart

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Chart, line chart

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R\_25 (proportion of immune: 25%) =

2.5942 2.6822 2.7122 2.7221 2.7252 2.7262 2.7266 2.7267 2.7267

R\_50 (proportion of immune: 50%) =

1.7295 1.7881 1.8082 1.8147 1.8168 1.8175 1.8177 1.8178 1.8178

R\_725 (proportion of immune: 72.5%) =

0.9512 0.9835 0.9945 0.9981 0.9992 0.9996 0.9997 0.9998 0.9998

R\_75 (proportion of immune: 75%) =

0.8647 0.8941 0.9041 0.9074 0.9084 0.9087 0.9089 0.9089 0.9089

The number of infectious people in each generation increases when the proportion of immune is 25% and 50%. The number of infectious people in each generation declines when the proportion of immune is 72.5% or greater than 72.5%. It is because the average number of secondary infectious persons resulting from each infectious person after a few

generations are about 2.7267, 1.8178, 0.9998, and 0.9089 when the proportion of immune is 25%, 50%, 72.5%, and 75% respectively. So, while R0 is bigger than 1 when the proportion of immune is 25% and 50%, R0 is smaller than 1 when the proportion of immune is 72.5% and 75%.